

Problem Definition

Problem 43. **Profit:** Because of a slump in the economy, a company finds that its annual profits have dropped from \$742,000 in 1998 to \$632,000 in 2000. If the profit follows an exponential pattern of decline, what is the expected profit for 2003? (Let $t = 0$ correspond to 1998.)

Solution Step 1:

Let's start by defining some variables that we will use in the development of the decline. We can use t to denote time and P to denote profit. An exponential model is of the form

$$P = Ce^{kt}$$

In order to model the profit as a function of time, we will need to compute both C and k .

Solution Step 2:

At the beginning of the time span of the model, we are given $t = 0$ corresponds to the year 1998 and the profit in 1998 is given by \$742,000. In terms of the model, this means

$$P = Ce^{k(0)} = C(1) = 742000$$

So, this gives us one of the two constants, $C = 742000$.

Solution Step 3:

We also know that in 2000 ($t = 2$) the profit has declined to value of \$632,000. In terms of the model, we find

$$P = (742000)e^{k(2)} = 632000$$

This relationship will allow us to determine the rate (of decline) constant k . This is done by doing some algebra. Dividing by 742000 gives

$$e^{2k} = \frac{632000}{742000} \approx 0.8518$$

Applying the natural logarithm to both sides of this result produces

$$\ln(e^{2k}) = 2k \approx \ln(0.8518) \approx -0.1605$$

Finally, dividing by 2 gives the following approximation for the rate of decline.

$$k \approx \ln(0.8518) \approx -0.0802$$

Solution Step 4:

With $C = 742000$ and $k = -0.0802$ the exponential decline model will be the following.

$$P = (742000)e^{(-0.0802)t}$$

In the year 2003, $t = 5$ and the predicted profit will be

$$P = (742000)e^{(-0.0802)(5)} \approx \$496806.10$$