

**Problem Definition**

Problem 39. Find the indefinite integral and check your result by differentiation.

$$\int \frac{1}{3x^4} dx$$

**Solution Step 1:**

To make the problem a bit easier we can rewrite the integrand to use the power rule. That is,

$$\frac{1}{3x^4} = \frac{1}{3} x^{-4}$$

**Solution Step 2:**

The power rule will allow us to compute this indefinite integral. The rule is

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C_1$$

where  $C_1$  is the constant of integration that must be included.

For the present problem  $n = -4$ , so

$$\begin{aligned} \int \frac{1}{3x^4} dx &= \int \frac{1}{3} x^{-4} dx \\ &= \frac{1}{3} \int x^{-4} dx \\ &= \frac{1}{3} \left( \frac{1}{-3} \right) x^{-3} + C \\ &= -\frac{1}{9} x^{-3} + C \\ &= -\frac{1}{9x^3} + C \end{aligned}$$

**Solution Step 3:**

Now, let's check the answer. We differentiate our result above with respect to  $x$ .

$$\begin{aligned}\frac{d}{dx} \left( -\frac{1}{9}x^{-3} + C \right) &= -\frac{d}{dx} \left( \frac{1}{9}x^{-3} \right) + \frac{d}{dx} (C) \\ &= -\frac{1}{9} \frac{d}{dx} (x^{-3}) + (0) \\ &= -\frac{1}{9}(-3)(x^{-4}) \\ &= \frac{1}{3} (x^{-4}) \\ &= \frac{1}{3x^4}\end{aligned}$$