Problem Definition

Problem 39. Use formal substitution as in Example 5 in the textbook to find the indefinite integral.

$$\int \frac{x}{x^2+25}dx$$

Solution Step 1:

The formal substitution should result in a new integral that is easier to compute. The main problem in the integrand is the quadratic polynomial in the denominator of the rational expression. One way to start is to substitute for this expression. The result is to choose

$$u = x^2 + 25$$

From this we can compute $du = 2xdx$. The integral can be rewritten in the following manner.

$$\begin{aligned} \int \frac{x}{x^2+25}dx &= \int \frac{1}{2}\left(\frac{2x}{x^2+25}\right)dx \\ &= \frac{1}{2}\int \frac{1}{x^2+25}(2xdx) \\ &= \frac{1}{2}\int \frac{1}{u}du \end{aligned}$$

Solution Step 2:

The transformed integral is a lot easier to compute.

$$\frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}ln|u| + C$$

Solution Step 3:

The last step is to transform the antiderivative back in terms of the original variables. This gives

$$\begin{aligned} \int \frac{x}{x^2+25}dx &= \frac{1}{2}ln|u| + C \\ &= \frac{1}{2}ln|x^2+25| + C \end{aligned}$$

Solution Step 4:

It is relatively easy to check our work in this problem. All we need to do is differentiate the general antiderivaive as follows.

$$\begin{aligned}
\frac{d}{dx}\left(\frac{1}{2}ln|x^2+25|+C\right) &= \frac{d}{dx}\left(\frac{1}{2}ln|x^2+25|\right)+\frac{d}{dx}(C) \\
&= \frac{1}{2}\frac{d}{dx}\left(ln|x^2+25|\right)+(0) \\
&= \frac{1}{2}\left(\frac{1}{x^2+25}\right)\frac{d}{dx}\left(x^2+25\right) \\
&= \frac{1}{2}\left(\frac{1}{x^2+25}\right)(2x) \\
&= \frac{x}{x^2+25}
\end{aligned}$$