Problem Definition

Problem 53. **Demand Function** Find the demand function $x = f(p)$ that satisfies the initial conditions.

$$\frac{dx}{dp} = -\frac{6000p}{(p^2 - 16)^{3/2}}$$

When $x = 5000$ we have $p = \$5$.

Solution Step 1:

To start, we need to compute the general antiderivative for the given rate of change of demand with respect to price. The integral we need to compute is

$$\int \frac{dx}{dp} dp = \int \left(-\frac{6000p}{(p^2 - 16)^{3/2}} \right) dp$$

To do this we will need to do a substitution. The idea is to substitute for the trouble spot in the integral which is the denominator. In particular, the expression in the power. So, we can define

$$u = p^2 - 16$$

The substitution requires $du = 2pdp$. We can rewrite the integral as follows.

$$\begin{aligned}
\int \left(-\frac{6000p}{(p^2 - 16)^{3/2}} \right) dp &= \int \left(-3000 \frac{2p}{(p^2 - 16)^{3/2}} \right) dp \\
&= -3000 \int \left(\frac{1}{(p^2 - 16)^{3/2}} \right) (2pdp) \\
&= -3000 \int \left(\frac{1}{u^{3/2}} \right) du \\
&= -3000 \int u^{-3/2} du
\end{aligned}$$

This is a much easier integral to work with.

Solution Step 2:

The general antiderivative is computed as follows.

$$\begin{aligned} -3000\int u^{-3/2}du &= -3000\left(\frac{1}{-1/2}u^{-1/2}\right)+C \\ &= 6000u^{-1/2}+C \end{aligned}$$

Solution Step 3:

For this problem we will want the antiderivative in terms of the original variables. This is done as follows.

$$x=\int \frac{dx}{dp}dp = 6000u^{-1/2}+C = 6000(p^2-16)^{-1/2}+C$$

Solution Step 1:

The last step in the process is to enforce the condition on the price and demand values given in the problem. When price is $p = \$5$, the demand, $x = 5000$, should be predicted by the model. So,

$$\begin{aligned} 5000 &= 6000(5^2-16)^{-1/2}+C \\ &= 6000\,\frac{1}{3}+C \\ &= 2000+C \end{aligned}$$

To make this work, $C = 3000$ and the model requires

$$x = 6000(p^2-16)^{-1/2}+3000 = \frac{6000}{\sqrt{p^2-16}}+3000$$