Problem Definition

Problem 21. Use the Log Rule to find the indefinite integral.

$$\int \frac{x^2}{x^3+1}\ dx$$

Solution Step 1:

The problem requires that we perform a bit of algebra to get the integral in the correct form. We start with

$$u = x^3 + 1$$

This is the entire expression in the denominator. This requires that we have

$$\frac{du}{dx} = 3x^2$$

The integral can be written as

$$\begin{aligned} \int \frac{x^2}{x^3+1}\ dx &= \int \frac{1}{3}\frac{3x^2}{x^3+1}\ dx \\ &= \frac{1}{3}\int \frac{du/dx}{u}\ dx \\ &= \frac{1}{3} ln|u| + C \end{aligned}$$

applying the generalized log rule for computing antiderivatives.

Solution Step 2:

The next step is to return to the original variables. The result is

$$\frac{1}{3}\ ln|u| + C = \frac{1}{3}\ ln|x^3+1| + C$$

Solution Step 3:

We can easily check the result by differentiating the antiderivative.

$$\begin{aligned}
\frac{d}{dx}\left(\frac{1}{3}\ ln|x^3+1|+C\right) &= \frac{d}{dx}\left(\frac{1}{3}\ ln|x^3+1|\right)+\frac{d}{dx}(C) \\
&= \frac{1}{3}\frac{d}{dx}\left(ln|x^3+1|\right)+(0) \\
&= \frac{1}{3}\left(\frac{1}{x^3+1}\right)\frac{d}{dx}\left(|x^3+1|\right) \\
&= \frac{1}{3}\left(\frac{1}{x^3+1}\right)(3x^2) \\
&= \frac{x^2}{x^3+1}
\end{aligned}$$