Problem Definition

Problem 45. Use any basic integration formulas to compute the following indefinite integral.

$$\int 4e^{2x-1}\,dx$$

Solution Step 1:

The problem requires that we perform a simple substitution. We start with

$$u = 2x - 1$$

The idea of this simple substitution is to transform to a simpler argument in the natural exponential. The substitution requires $du = 2dx$. The integral can be rewritten as follows.

$$\begin{aligned} \int 4e^{2x-1}\,dx &= \int 2e^{2x-1}(2dx) \\ &= \int 2e^{u}du \\ &= 2\int e^{u}du \end{aligned}$$

Solution Step 2:

The indefinite can be computed using the following.

$$2\int e^{u}du = 2e^{u} + C$$

where C is the constant of integration.

Solution Step 3:

The last step in the process is to get the antiderivative back in terms of the original variables. This means

$$\int 4e^{2x-1}\,dx = 2e^{u} + C = 2e^{2x-1} + C$$

Solution Step 4:

The result can be checked by differentiating our answer to make sure that the antiderivative is correct. This is done as follows.

$$\begin{aligned}
\frac{d}{dx}\left(2e^{2x-1}+C\right) &= \frac{d}{dx}\left(2e^{2x-1}\right)+\frac{d}{dx}(C) \\
&= 2\frac{d}{dx}\left(e^{2x-1}\right)+(0) \\
&= 2e^{2x-1}\frac{d}{dx}(2x-1) \\
&= 2e^{2x-1}(2) \\
&= 4e^{2x-1}
\end{aligned}$$