

Problem Definition

Problem 25. Evaluate the definite integral.

$$\int_0^3 (x - 2)^3 dx$$

Solution Step 1:

Let's start by computing the antiderivative

$$\int (x - 2)^3 dx$$

using the substitution $u = x - 2$ which requires that $du = dx$. The indefinite integral is computed using

$$\int (x - 2)^3 dx = \int u^3 du = \frac{1}{4} u^4 + C$$

Transforming back to the original variable the antiderivative is

$$\int (x - 2)^3 dx = \frac{1}{4} (x - 2)^4 + C$$

Solution Step 2:

Now, we need to compute the definite integral. Since any antiderivative will do we can choose $C = 0$ in the antiderivative we computed in the previous step. The value of the definite integral is

$$\begin{aligned} \int_0^3 (x - 2)^3 dx &= \left(\frac{1}{4} (x - 2)^4 \right) \Big|_0^3 \\ &= \left(\frac{1}{4} (3 - 2)^4 \right) - \left(\frac{1}{4} (0 - 2)^4 \right) \\ &= \frac{1}{4} (1) - \frac{1}{4} (16) \\ &= -\frac{15}{4} \end{aligned}$$