Problem Definition

Problem 37. Evaluate the definite integral.

$$\int_1^3 \frac{e^{3/x}}{x^2} dx$$

Solution Step 1:

Let's start by computing the antiderivative

$$\int \frac{e^{3/x}}{x^2} dx$$

using the substitution $u = 3/x$ which requires that

$$du = -\frac{3}{x^2}\ dx$$

The indefinite integral is computed using

$$\begin{aligned} \int \frac{e^{3/x}}{x^2} dx &= \int \frac{1}{(-3)} e^{3/x}\ \frac{-3}{x^2}\ dx \\ &= -\frac{1}{3} \int e^u\ du \\ &= -\frac{1}{3} e^u + C \end{aligned}$$

Transforming back to the original variable the antiderivative is

$$\int \frac{e^{3/x}}{x^2} dx = -\frac{1}{3} e^{3/x} + C$$

Solution Step 2:

Now, we need to compute the definite integral. Since any antiderivative will do we can choose $C = 0$ in the antiderivative we computed in the previous step. The value of the definite integral is

$$
\begin{aligned}
\int_1^3 \frac{e^{3/x}}{x^2} dx &= \left(-\frac{1}{3}e^{3/x}\right)\Big|_1^3 \\
&= \left(-\frac{1}{3}e^{3/(3)}\right) \\
-\left(-\frac{1}{3}e^{3/(1)}\right) \\
&= \left(-\frac{1}{3}e^{1} + \frac{1}{3}e^{3}\right) \\
&= \frac{1}{3}e^{1}\left(e^{2} - 1\right) \\
&= \approx 5.789
\end{aligned}
$$