Problem Definition

Problem 91. **Compound Interest** A deposit of \$2250 is made in a savings account at an annual interest rate of 12%, compounded continuously. Find the average balance during the first five years.

Solution Step 1:

The first step is to write down the model for the balance in the investment. The model is of the form

$$P = Ae^{rt}$$

For this problem, the initial investment is $A = \$2250$, and $r = 0.12$. The form is

$$P = 2250e^{(0.12)t}$$

Solution Step 2:

To compute the average value of the investement over a five year period will require the computation of the following definite integral.

$$\begin{aligned}
\overline{P} &= \frac{1}{5-0}\int_0^5 (2250)e^{(0.12)t} \\
&= \frac{2250}{5}\int_0^5 e^{(0.12)t} \\
&= 450\int_0^5 e^{(0.12)t} \\
&= 450\left(\frac{1}{0.12}\right)e^{(0.12)t}\Big|_0^5 \\
&= \frac{450}{0.12}\left(e^{(0.12)(5)} - e^{(0.12)(0)}\right) \\
&\approx (3750)\,(0.822119) \\
&\approx 3082.94
\end{aligned}$$

The average balance over this five year period is approximately \$3082.94. There may be some discrepencies due to the number of digits kept in the calculations. The variable $\overline{P}$ is used to denote the average value of the profit function.