Problem Definition

Problem 23. Sketch the region between the graphs of the functions and compute the area of this region.

$$y = \frac{8}{x}, \quad y = x^2, \quad y = 0, \quad x = 1, \quad x = 4$$

Solution Step 1:

Use a graphing utility or graph the functions to determine the region for which the area is wanted. Without the sketch we need to find out which function produces larger values in the region and where the intersections of the graphs occur.

Solution Step 2:

The area in question is the area above the x axise between $x = 1$ and $x = 4$ below the graphs of the two functions. The next deal is to determine where the two curves above the x axis intersect. The intersection occurs when $y = 8/x$ and $y = x^2$. This means

$$\frac{8}{x} = x^2$$

or

$$8 = x^3$$

This is true for $x = 2$. So, the function $y = x^2$ will be used as the upper boundary of the region from $x = 1$ to $x = 2$ and the function $y = 8/x$ will be used as the upper boundary from $x = 2$ to $x = 4$.

Solution Step 3:

The area is computed using the integral

$$\begin{aligned} \text{Area} &= \frac{x^3}{3}\Big|_1^2 + 8\ ln(x)\Big|_2^4 \\ &= \left(\frac{2^3}{3} - \frac{1^3}{3}\right) + (8\ ln(4) - 8\ ln(2)) \end{aligned}$$

$$
\begin{aligned}
&= \frac{7}{3} + 8 \ \Big(ln(2^2) - ln(2)\Big) \\
&= \frac{7}{3} + 8 \ (2 \ ln(2) - ln(2)) \\
&= \frac{7}{3} + 8 \ (2 - 1) \, ln(2) \\
&= \frac{7}{3} + 8 \ ln(2) \\
&\approx 7.8785
\end{aligned}
$$

The value in the second to the last line is the solution in given in the back of the book.