Problem Definition

Problem 15. Find the indefinite integral.

$$\int \frac{e^{3x}}{1-e^{3x}}\ dx$$

Solution Step 1:

In this section we are trying to simplify the indefinite integral using some sort of substitution. To simplify things we will try substituting for the denominator. That is,

$$u = 1 - e^{3x}$$

Then

$$du = 0 - e^{3x}(3) = -3e^{3x}$$

Solution Step 2:

The indefinite integral can be rewritten and computed as

$$\begin{aligned}
\int \frac{e^{3x}}{1-e^{3x}}\ dx &= \int \frac{1}{(-3)}\frac{(-3)e^{3x}}{1-e^{3x}}\ dx \\
&= -\frac{1}{3}\int \frac{1}{1-e^{3x}}\ (-3e^{3x}dx) \\
&= -\frac{1}{3}\int \frac{1}{u}\ du \\
&= -\frac{1}{3}ln|u| + C
\end{aligned}$$

Solution Step 3:

The last thing to do is to substitute back in with the original variables.

$$-\frac{1}{3}ln|u| + C = -\frac{1}{3}ln|1-e^{3x}| + C$$

Solution Step 4:

We can check our indefinite integra using differentiation.

$$\begin{aligned}
\frac{d}{dx}\left(-\frac{1}{3}ln|1-e^{3x}|+C\right) &= \frac{d}{dx}\left(-\frac{1}{3}ln|1-e^{3x}|\right)+\frac{d}{dx}(C) \\
&= -\frac{1}{3}\frac{d}{dx}\left(ln|1-e^{3x}|\right)+(0) \\
&= -\frac{1}{3}\left(\frac{1}{1-e^{3x}}\right)\frac{d}{dx}(1-e^{3x}) \\
&= -\frac{1}{3}\left(\frac{1}{1-e^{3x}}\right)(-3e^{3x}) \\
&= \left(\frac{1}{1-e^{3x}}\right)e^{3x} \\
&= \frac{e^{3x}}{1-e^{3x}}
\end{aligned}$$