Problem Definition

Problem 33. Evaluate the indefinite integral.

$$\int \frac{2\sqrt{t}+1}{t}\ dt$$

Solution Step 1:

We want to substitute to remove complicated expressions. However, there is an easier way. That is, divide the t in the denominator into the terms to form

$$\begin{aligned}
\int \frac{2\sqrt{t}+1}{t}\ dt &= \int \frac{2t^{1/2}+1}{t}\ dt \\
&= \int 2t^{-1/2} + t{-}1\ dt \\
&= 2\frac{t^{1/2}}{1/2} + ln|t| + C \\
&= 4t^{1/2} + ln|t| + C \\
&= 4\sqrt{t} + ln|t| + C
\end{aligned}$$

Solution Step 1:

The solution can be checked by differentiating the indefinite integral to make sure we get back to the original integrand.

$$\begin{aligned}
\frac{d}{dt}\left(4\sqrt{t} + ln|t| + C\right) &= \frac{d}{dt}\left(4\sqrt{t} + ln|t| + C\right) \\
&= 4\frac{d}{dt}\left(t^{1/2}\right) + \frac{d}{dt}\left(ln|t|\right) + \frac{d}{dt}\left(C\right) \\
&= 4\left(\frac{1}{2}t^{-1/2}\right) + \frac{1}{t} + (0) \\
&= 2t^{-1/2} + \frac{1}{t} \\
&= \frac{2t^{1/2}+1}{t} = \frac{2\sqrt{t}+1}{t}
\end{aligned}$$