Problem Definition

Problem 65. **Meterology** During a two week period in March in a small town new Lake Erie, the measurable snowfall S (in inches) on the ground can be modeled by

$$S(t) = t\sqrt{14-t} \qquad 0 \leq t \leq 14$$

where t represents time.

(a) Use a graphing utility to graph the function.

(b) Find the average amount of snow on the ground during the two-week period.

(c) Find the total snowfall over the two-week period.

Solution Step 1:

Use a graphing utility to graph the function.

Solution Step 2:

To find the average amount of snowfall during the two week period we can compute the definite integral.

$$\frac{1}{14-0}\int_0^{14}\left(t\sqrt{14-t}\right)\,dt \;=\; \frac{1}{14}\int_0^{14}\left(t\sqrt{14-t}\right)\,dt$$

To simplify the integration we can make the substitution

$$u = 14 - t$$

with $du = -dt$. We will need to replace the factor of t in the integrand. This can be done using

$$t = 14 - u$$

Also, we can transform the limits of integration as follows.

t = 0	u = 14
t = 14	u = 0

Solution Step 3:

The definite integral can be evaluated using the substitutions above.

$$\begin{aligned}
\frac{1}{14}\int_0^{14}\left(t\sqrt{14-t}\right)\,dt &= \frac{1}{14}\int_{14}^{0}\left((14-u)\sqrt{u}\right)\,(-du)\\
&= -\frac{1}{14}\int_{14}^{0}\left((14u^{1/2}-u^{3/2})\right)\,du\\
&= \frac{1}{14}\int_{0}^{14}\left((14u^{1/2}-u^{3/2})\right)\,du\\
&= \frac{1}{14}\left((\frac{28}{3}\,u^{3/2}-\frac{2}{5}\,u^{5/2})\right)|_0^{14}\\
&= \frac{1}{14}\left((\frac{28}{3}\,(14)^{3/2}-\frac{2}{5}\,(14)^{5/2})\right)\\
&\approx 13.97
\end{aligned}$$

The average snowfall is about 13.97 inches.

Solution Step 4:

The total snowfall for the same time period is the integral value before dividing by the number of days. That is, the total snowfall is approximately $14 \times 13.97 = 195.57$ inches. That is the total is equal to the average multiplied by the number of time units or days in this case.