

Problem Definition

Problem 29. Evaluate the definite integral.

$$\int_0^1 x^2 e^x dx$$

Solution Step 1:

For this problem, we can apply the following definitions for integration by parts.

$$\begin{aligned} u &= x^2 & du &= 2x dx \\ dv &= e^x dx & v &= e^x \end{aligned}$$

The application of these definitions produces

$$\begin{aligned} \int_0^1 x^2 e^x dx &= (x^2 e^x) \Big|_0^1 - \int_0^1 2x e^x dx \\ &= ((1)^2 e(1)) - ((0)^2 e(0)) - \int_0^1 2x e^x dx \\ &= e - 2 \int_0^1 x e^x dx \end{aligned}$$

Solution Step 2:

To complete the work on this problem, we still need to do an additional integration by parts. For this step we can use

$$\begin{aligned} u &= x & du &= dx \\ dv &= e^x dx & v &= e^x \end{aligned}$$

The application of these definitions produces

$$\begin{aligned} e - 2 \int_0^1 x e^x dx &= e - 2 \left(x e^x \Big|_0^1 - \int_0^1 e^x dx \right) \\ &= e - 2 \left(((1)e(1) - (0)e(0)) - e^x \Big|_0^1 \right) \\ &= e - 2 \left(e - (e(1) - e(0)) \right) \\ &= e - 2(e - (e - 1)) \\ &= e - 2e + 2e - 2 \\ &= e - 2 \approx 0.718 \end{aligned}$$