

Problem Definition

Problem 41. Find the indefinite integral using each specified method. The write a brief statement explaining which method you prefer.

$$\int \frac{x}{\sqrt{4+5x}} dx$$

- (a) By parts, letting $dv = \frac{1}{\sqrt{4+5x}}$.
(b) By parts, letting $u = \sqrt{4+5x}$.

Solution Step 1:

The integration by parts using $dv = \frac{1}{\sqrt{4+5x}}$ means that we need to let $u = x$. This means

$$\begin{aligned} u &= x & du &= dx \\ dv &= (4+5x)^{-1/2} & v &= 2/5(4+5x)^{1/2} \end{aligned}$$

The integral becomes

$$\begin{aligned} \int \frac{x}{\sqrt{4+5x}} dx &= \frac{2}{5} x\sqrt{4+5x} - \int \frac{2}{5}(4+5x)^{1/2} dx \\ &= \frac{2}{5} x\sqrt{4+5x} - \frac{2}{5} \int (4+5x)^{1/2} dx \\ &= \frac{2}{5} x\sqrt{4+5x} - \frac{2}{5} \left(\frac{2}{15} (4+5x)^{3/2} \right) + C \\ &= \frac{2}{5} x\sqrt{4+5x} - \frac{4}{75} (4+5x)^{3/2} + C \\ &= \frac{2}{75} \sqrt{4+5x} (5x-8) + C \end{aligned}$$

Solution Step 2:

For the second version of the problem, we use $u = \sqrt{4+5x}$. We need to do a bit more work on the integral before proceeding. The integral can be rewritten as

$$\int \frac{x}{\sqrt{4+5x}} dx = \int \frac{x}{4+5x} \sqrt{4+5x} dx$$

The table of equations for this version of integration by parts gives

$$\begin{aligned}u &= \sqrt{4 + 5x} & du &= \frac{5}{2} (4 + 5x)^{-1/2} dx \\dv &= \frac{x}{4+5x} & v &= \frac{1}{25}(4 + 5x + 4\ln|4 + 5x|)\end{aligned}$$

The integral becomes

$$\begin{aligned}\int \frac{x}{\sqrt{4 + 5x}} dx &= \sqrt{4 + 5x} \left(\frac{1}{25} (4 + 5x + 4\ln|4 + 5x|) \right) \\&\quad - \int \frac{1}{25} (4 + 5x + 4\ln|4 + 5x|) \left(\frac{5}{2} \right) (4 + 5x)^{-1/2} dx\end{aligned}$$

The integral will be the same. However, the amount of work seems to be a lot more for this problem.