Problem Definition

Problem 61. **Learning Theory** A model of the ability M of a child to memorize, measured on a scale from 00 to 10, is

$$M = 1 + 1.6\ t\ ln(t), \qquad 0 < t \leq 4$$

where t is the child's age in years. Find the average value of this model between

(a) the child's first and second birthdays, and

(b) the child's third and fourth birthdays.

Solution Step 1:

To make the work go a bit faster below, let's compute the indefinite integral ofthe logarithm term. That is,

$$\int t\ ln(t)\ dt$$

This can be done by an application of integration by parts. The definitions are

$$\begin{array}{ll} u = ln(t) & du = t^{-1}dt \\ dv = tdt & v = \frac{1}{2}t^2 \end{array}$$

So, the integration proceeds as follows:

$$\begin{aligned} \int t\ ln(t)\ dt &= \frac{1}{2}\ t^2 ln(t) - \int t^2 \left(\frac{1}{2}t^2\right)\ dt \\ &= \frac{1}{2}\ t^2 ln(t) - \frac{1}{2}\int t\ dt \\ &= \frac{1}{2}\ t^2 ln(t) - \frac{1}{4}t^2 + C \\ &= \frac{1}{2}\ t^2 (ln(t) - \frac{1}{2}) + C \end{aligned}$$

For our problem, we can choose $C = 0$ since we only need to use this to evaluate a definite integral.

Solution Step 2:

We can take the calculations from the last step a bit further by computing the antiderivative for the function of interest as follows.

$$\begin{aligned}
\int M\ dt &= \int (1 + 1.6\ t\ ln(t))\ \ dt \\
&= \int (1)\ \ dt + (1.6) \int (t\ ln(t))\ \ dt \\
&= t + (1.6) \left(\frac{1}{2}\ t^2 (ln(t) - \frac{1}{2}) \right) + C \\
&= t + (0.8) \left(t^2 (ln(t) - \frac{1}{2}) \right) + C \\
&= t + (0.8) t^2 \left(ln(t) - \frac{1}{2} \right)
\end{aligned}$$

where again, we have chosen to set the integration constant to zero for the calculation of the definite integrals below.

Solution Step 3:

Now, for part (a) we can compute the value of the definite integral for the second year of the child's life. That is,

$$\begin{aligned}
\frac{1}{2-1} \int_1^2 M\ dt &= t + (0.8) t^2 \left(ln(t) - \frac{1}{2} \right) |_1^2 \\
&= (2) + (0.8)(2)^2 \left(ln((2)) - \frac{1}{2} \right) \\
&\quad -(1) + (0.8)(1)^2 \left(ln((1)) - \frac{1}{2} \right) \\
&\approx 2.018
\end{aligned}$$

Solution Step 3:

Now, for part (b) we need to compute another definite integral over a child's third year of life.

$$\begin{aligned}
\frac{1}{4-3} \int_3^4 M\ dt &= t + (0.8) t^2 \left(ln(t) - \frac{1}{2} \right) |_1^2 \\
&= (4) + (0.8)(4)^2 \left(ln((4)) - \frac{1}{2} \right) \\
&\quad -(3) + (0.8)(3)^2 \left(ln((3)) - \frac{1}{2} \right) \\
&\approx 8.035
\end{aligned}$$