Problem Definition

Problem 21. Find the indefinite integral.

$$\int \frac{3}{x^2 + x - 2}\, dx$$

Solution Step 1:

In this case the degree of the polynomial in the denominator (two) is greater than the degree of the polynomial in the numerator (zero). So, we do not need to do a long division on the rational expression. We can go directly to the partial fraction decomposition. The form is given by

$$\frac{3}{x^2 + x - 2} = \frac{3}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

Clearing fractions by multiplying the equation by $(x+2)(x-1)$ gives the equation

$$3 = A(x-1) + B(x+2)$$

The coefficients A and B can be obtained using specific values of x. If we choose $x = 1$ the equation becomes

$$3 = A(1-1) + B(1+2)$$

or

$$3 = 3B \quad \rightarrow \quad B = 1$$

If we choose $x = -2$, the equation for A and B becomes

$$3 = A(-2-1) + B(-2+2)$$

or

$$3 = -3A \quad \rightarrow \quad A = -1$$

The final partial fraction decomposition is

$$\frac{3}{x^2 + x - 2} = \frac{-1}{x+2} + \frac{1}{x-1} = \frac{1}{x-1} - \frac{1}{x+2}$$

Solution Step 2:

Now we can compute the indefinite integral as follows.

$$\begin{aligned}
\int \frac{3}{x^2+x-2}\,dx &= \int \left(\frac{1}{x-1} - \frac{1}{x+2}\right) \\
&= \int \left(\frac{1}{x-1}\right) - \int \left(\frac{1}{x+2}\right) \\
&= ln|x-1| - ln|x+2| + C \\
&= ln\frac{|x-1|}{|x+2|} + C \\
&= ln\left|\frac{x-1}{x+2}\right| + C
\end{aligned}$$