Problem Definition

Problem 39. **Present Value** A business is expected to yield a continuous flow of profit at the rate of \$500,000 per year. If money will earn interest at the nominal rate of 9% per year compounded continuously, what is the present value of the business (a) for 20 years and (b) forever? (Present Value is defined in Section 6.2 of the text.)

Solution Step 1:

The present value calculation for part (a) is

$$\begin{aligned}
\text{Present Value} &= \int_0^{20} (500,000)e^{(-0.09)t}\ dt \\
&= (500,000)\int_0^{20} e^{(-0.09)t}\ dt \\
&= (500,000)\frac{e^{(-0.09)t}}{-0.09}|0^{20} \\
&= \frac{500,000}{-0.09}(e^{(-0.09)(20)} - e^{(-0.09)(0)}) \\
&\approx -(5,555,555.56)(e^{(-1.8)} - 1) \\
&\approx 4,637,228
\end{aligned}$$

That's a lot of money.

Solution Step 2:

For part (b) we are trying to find the present value of the company for the foreseeable future. The model is to take t out to infinity.

$$\begin{aligned}
\text{Present Value} &= \int_0^{\infty} (500,000)e^{(-0.09)t}\ dt \\
&= (500,000)\lim_{b\to\infty}\int_0^{b} e^{(-0.09)t}\ dt \\
&= (500,000)\lim_{b\to\infty}\frac{e^{(-0.09)t}}{-0.09}|0^{b} \\
&= \frac{500,000}{-0.09}\lim_{b\to\infty}(e^{(-0.09)b} - e^{(-0.09)(0)}) \\
&\approx -(5,555,555.56)(0 - 1) \\
&\approx 5,555,556
\end{aligned}$$

This is due to the fact that the exponential approaches zero as the the argument tends to $-\infty$.