

### Problem Definition

Problem 33. For the function

$$w = \sqrt{x^2 + y^2 + z^2}$$

find  $w_x$ ,  $w_y$ , and  $w_z$  at the point  $(2, -1, 2)$ .

### Solution Step 1:

There are three derivatives we need to compute. The partial derivative of  $w$  with respect to  $x$  is

$$\begin{aligned} w_x &= \frac{\partial}{\partial x}(x^2 + y^2 + z^2)^{1/2} \\ &= \frac{1}{2}(x^2 + y^2 + z^2)^{-1/2} \frac{\partial}{\partial x}(x^2) \\ &= \frac{1}{2}(x^2 + y^2 + z^2)^{-1/2}(2x) \\ &= \frac{x}{\sqrt{x^2 + y^2 + z^2}} \end{aligned}$$

Similarly,

$$\begin{aligned} w_y &= \frac{y}{\sqrt{x^2 + y^2 + z^2}} \\ w_z &= \frac{z}{\sqrt{x^2 + y^2 + z^2}} \end{aligned}$$

### Solution Step 2:

Now, we can evaluate each of the derivatives at the point  $(x, y, z) = (2, -1, 2)$ .

$$\begin{aligned} w_x(2, -1, 2) &= \frac{2}{\sqrt{(2)^2 + (-1)^2 + (2)^2}} = \frac{2}{3} \\ w_y(2, -1, 2) &= \frac{-1}{\sqrt{(2)^2 + (-1)^2 + (2)^2}} = \frac{-1}{3} \\ w_z(2, -1, 2) &= \frac{2}{\sqrt{(2)^2 + (-1)^2 + (2)^2}} = \frac{2}{3} \end{aligned}$$